

Relating Peak Particle Velocity and Acceleration to Moment Magnitude in Passive (Micro-)Seismic Monitoring

Introduction

Passive Seismic Monitoring (PSM) systems are typically designed to detect acoustic events within a pre-defined Moment Magnitude range. To achieve this objective the receivers used in the PCM system must be able to cover this Moment Magnitude range as there is a direct relationship between the Moment Magnitude and the sensor's measurement range as well as resolution. Seismic sensors are typically designed to measure either particle velocity (geophones) or particle acceleration (accelerometers); therefore, the Moment Magnitude range of a PSM system can be ascertained by establishing an analytical relationship between Moment Magnitude and peak particle velocity (PPV) and acceleration (PPA). This technical note outlines this relationship.

Brune's Far-Field Displacement Pulse

The utilization of Brune's Far-Field displacement pulse is fundamental for establishing an analytical relationship between Moment Magnitude and PPV and PPA. Brune's Far-Field displacement pulse is derived based upon kinematic or quasidynamic dislocation models, which predict that the far-field displacement spectrum should remain constant at low frequencies and become inversely proportional to some power of frequency at higher frequencies (Brune, 1970 and 1971). The three independent parameters specifying the far-field displacement spectrum are the low-frequency level, the corner frequency (defined as the intersection of the low- and high-frequency asymptotes), and the slope coefficient controlling the rate of high-frequency decay of the spectrum. Brune's Far-Field displacement pulse can then be described as follows:

$$u(t) = \Omega_0 t \omega_0^2 H(t) e^{-t\omega_0} \quad (1)$$

where $H(t)$ is the Heaviside function and Ω_0 and ω_0 are the displacement spectra plateau and corner frequency, respectively. These two variables are derived from the displacement spectrum as illustrated in Figure 1, which shows a typical seismic displacement spectrum which is analytically defined as the Fourier transform of Brune's Far-Field pulse.

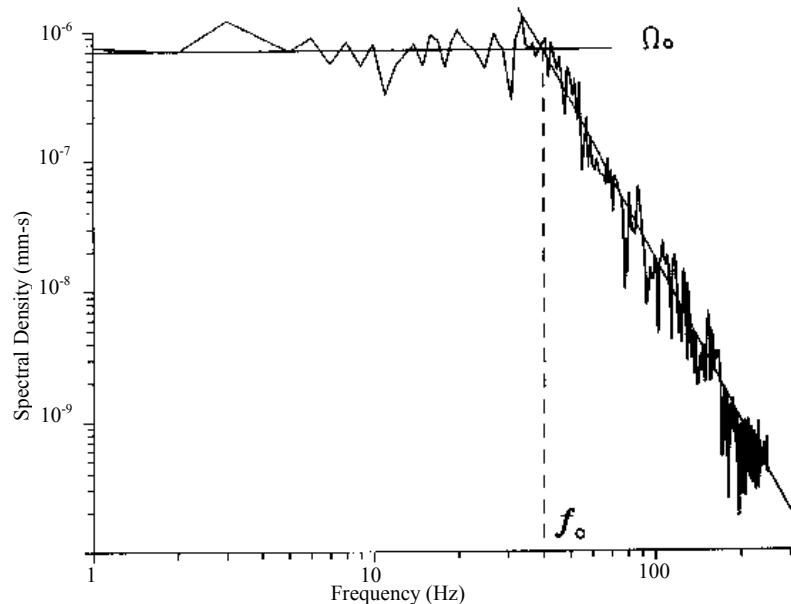


Figure 1. Illustration of Brune's Far-Field pulse in the frequency domain (Log₁₀ - Log₁₀ plot).

The Fourier transform of equation (1) is given as either

$$|U(\omega)| = \frac{\Omega_0}{1 + \left(\frac{\omega}{\omega_0}\right)^2} \quad (2a)$$

if it is assumed that the displacement spectrum has been corrected for attenuation, or if not as

$$|U(\omega)| = \frac{\Omega_0 e^{-\omega R/2vQ}}{1 + \left(\frac{\omega}{\omega_0}\right)^2} \quad (2b)$$

where R is the source-sensor distance, v is the velocity and Q is the quality factor of the P or S wave.

It is clear from equations (2) that the absorption of the medium can be estimated from the displacement spectrum.

The PPV based upon Brune's Far-Field displacement pulse is then obtained by differentiating equation (1):

$$u'(t) = \Omega_0 \omega_0^2 [H(t)e^{-t\omega_0} - \omega_0 t H(t)e^{-t\omega_0} + t \delta(t)e^{-t\omega_0}] \quad (3)$$

It is evident from equation (3) that $u'(t)$ is maximum at $t = 0$; therefore, the PPV for a Brune's Far-Field source wave is

$$PPV = \Omega_0 \omega_0^2 \quad (4)$$

The PPA is obtained by differentiating equation (3) and obtaining the maximum value at $t = 0$. The PPA value for a Brune's Far-Field displacement pulse is given as¹

$$|PPA| = |\Omega_0 \omega_0^2 (1 - 2\omega_0)| = |PPV (1 - 2\omega_0)| \quad (5)$$

Seismic Moment

The Seismic Moment M_0 is a very reliable and useful measure of the strength of a seismic event, and is defined as follows in terms of parameters of the double-couple shear dislocation source model:

$$M_0 = G \bar{u} A \quad (6)$$

where G is the shear modulus, \bar{u} is the average displacement across the fault, and A is the fault area.

*¹ Note: $t\delta'(t) = -\delta(t)$

For small seismic events, such as mine tremors, the displacement spectra parameter Ω_0 is commonly used for reliable determination of the Seismic Moment. The low-frequency level Ω_0 of the far-field displacement spectrum of P or S waves is directly related to the Seismic Moment as follows

$$M_o = 4\pi\rho_0v_0^3R\Omega_0/F_cR_c S_c \quad (7)$$

with

- ρ_0 the density of source material;
- v_0 either P-wave velocity or the S-wave velocity at the source;
- R the distance between the source and the receiver;
- F_c factor to account for the radiation of either P- or S-waves
- R_c factor to account for the free-surface amplification of either P- or S-wave amplitudes; the free-surface effect is expressed by a factor of 2 for SH waves, whereas for SV and P waves it is a function of the angle of incidence and frequency and has to be specified; however, the free-surface correction is generally neglected (i.e., set to one) for sensors located in underground mines, especially for those located in boreholes (Gibowicz and Kijko, 1994)
- S_c the site correction for either P or S waves; once again site corrections are often neglected, although near-surface site response is an important factor in source studies (Gibowicz and Kijko, 1994).

Moment-Magnitude

The Moment-Magnitude relations are empirically written as a linear relation between the logarithmic value of the Seismic Moment M_0 and the Magnitude M_n

$$\log M_0 = aM_n + b \quad (8)$$

where a and b are constants.

The Moment Magnitude has been defined by Hanks and Kanamori (1979) as:

$$M = 2/3 \text{ Log} M_0 - 6.0 \quad (9)$$

Tables can now be generated showing the relationship between the Moment Magnitude and PPV and PPA by utilizing equations (4), (5), (7) and (9), the attenuation corrected displacement spectra and typical in-situ parameters for a site under investigation.

Example

Consider that an investigator is designing a PSM system which is to monitor out to $R = 60\text{m}$ from a vertically installed array of seismic sensors within a borehole. The following in-situ parameters exist for an array of sensors installed within a borehole:

- $\rho_0 = 2400 \text{ kg/m}^3$;
- $v_0 = 2100 \text{ m/s}$ (P-wave velocity);
- $R = 60 \text{ m}$;
- $F_c = 0.52$;
- $R_c = 1$;
- $S_c = 1$;
- $M_{\min} = -3.0$
- $M_{\max} = 1.5$

Table 1 then relates the Moment Magnitude to corresponding PPV and PPA. It is hereby assumed that a typical P-wave corner frequency of 95Hz exists for equations (4) and (5) (i.e., $PPV = 3.56 \times 10^5 \Omega_0$ and $|PPA| = 1192 \times PPV$). Attenuated PPV and PPA values are also shown; these were calculated by applying equation (2b) and utilizing an average plateau frequency of 10 Hz and a P wave Q value of 40 1/dB or 4.6 1/Np ($e^{-\omega R/2vQ} = e^{-0.195} = 0.823$).

Table 1. Moment Magnitude with corresponding PPV and PPA values at 60m.

Moment Magnitude	Ω_0 [m-s]	PPV [m/s]	PPV _{attenuated} [m/s]	PPA [m/s]	PPA _{attenuated} [m/s]
-4.0	3.1×10^{-14}	1.1×10^{-08}	9.1×10^{-09}	1.3×10^{-05}	1.1×10^{-05}
-3.5	1.74×10^{-13}	6.2×10^{-08}	5.1×10^{-08}	7.4×10^{-05}	6.1×10^{-05}
-3.0	9.8×10^{-13}	3.5×10^{-07}	2.9×10^{-07}	4.1×10^{-04}	3.4×10^{-04}
-2.5	5.5×10^{-12}	2.0×10^{-06}	1.6×10^{-06}	2.3×10^{-03}	1.9×10^{-03}
-2.0	3.1×10^{-11}	1.1×10^{-05}	9.1×10^{-06}	1.3×10^{-02}	1.1×10^{-02}
-1.5	1.7×10^{-10}	6.2×10^{-05}	5.1×10^{-05}	7.4×10^{-02}	6.1×10^{-02}
-1.0	9.8×10^{-10}	3.5×10^{-04}	2.9×10^{-04}	4.2×10^{-01}	3.4×10^{-01}
-.05	2.6×10^{-08}	9.3×10^{-03}	7.7×10^{-03}	11.1	9.1
0	3.1×10^{-08}	1.1×10^{-02}	9.1×10^{-03}	13.2	10.8
0.05	3.7×10^{-08}	1.3×10^{-02}	1.1×10^{-02}	15.7	12.9
1.0	9.8×10^{-07}	3.5×10^{-01}	2.9×10^{-01}	417	343
1.5	5.5×10^{-06}	2.0	1.6	2344	1929
2.0	3.1×10^{-05}	11.0	9.1	1.3×10^{04}	1.1×10^{04}
2.5	1.7×10^{-04}	62	51	7.4×10^{04}	6.1×10^{04}
3.0	9.8×10^{-04}	349	288	4.2×10^{05}	3.4×10^{05}
3.5	5.5×10^{-03}	1965	1617	2.3×10^{06}	1.9×10^{06}
4.0	3.1×10^{-02}	11049	9093	1.3×10^{07}	1.1×10^{07}

The attenuated PPV for the lower and upper limit of the Moment is 2.9×10^{-7} and 1.6 respectively (and the seismic sensor should be able to measure these velocities). The amplitude ratio is $1.6/2.9 \times 10^{-7}$ or 135 dB. This means that a 24 bit resolution is required, as this level exceeds the upper limit for a 20 bit resolution (i.e. 120 dB).

Conclusion

Typical PSM systems are designed to acquire acoustic events within a predefined Moment Magnitude range. From the analysis procedures and results outlined in this technical note an investigator can readily select the most appropriate type of sensor. In addition, the A/D bit resolution can also be ascertained from the calculated resolution range of PPV and PPA values.

Erick Baziw
Gerald Verbeek

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